A Model-theoretic Account for Structural Realism

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August 21, 2014

Word count: 9856

obtained via: (http://app.uio.no/ifi/texcount/online.php)

Abstract

In this paper I will give a model-theoretic account for structural realism. I will introduce the so-called 'hierarchy of models' which can be used to formally analyse the structure shared by a 'high-level theory' and the 'phenomena'. I will show that structural empiricism (as characterized by Bueno (1997, 2000) and Van Fraassen (1980)) cannot be accommodated in the hierarchy of models and show that this is ultimately due to the absence of a justified account for the structure of the 'phenomena'. Next I will introduce Bertrand Russell's causal theory of perception and show that this theory can account for the structure of the 'phenomena'. Then I will show how, based on this account, structural empiricism can be accommodated in the hierarchy of models. After that, I will show that, by the addition of a layer called 'external world' to the hierarchy of models, we can accommodate structural realism in the thus obtained 'extended hierarchy of models'. This will provide us with the notion of a 'high-level theory' being 'structurally correct', which means that the 'external world' can be embedded into a 'high-level theory'. The extended hierarchy of models allows us to formally analyse certain relations between structural empiricism and structural realism. Based on these analyses we will see in what way structural realism can be considered to be an extension of structural empiricism. Finally, I will show how the extended hierarchy of models can be extended even further to account for the 'problem of theory-change' (Bueno, 2000).

Contents

INTRODUCTION

"Structural realism is considered by many realists and antirealists alike as the most defensible form of scientific realism" (Ladyman, 2014). 'Scientific realism' is the view that "we ought to believe in the unobservable entities posited by our most successful scientific theories" (Ladyman, 2014). It is widely held that the most powerful argument in favour of scientific realism is the so-called 'no-miracles augment', according to which the success of science would be a miracle if scientific theories were not at least approximately true descriptions of the world. Arguably, the most powerful arguments against scientific realism are grounded in the history of radical theory-change in science. The best-known of these arguments is the so-called 'pessimistic meta-induction', according to which reflection on the abandonment of theories in the history of science motivates the expectation that our best scientific theories will themselves be abandoned, and hence that we should not assent to them (Ladyman, 2014).

'Structural realism' was introduced into contemporary philosophy of science by John Worrall (1989) as a way to break the impasse that results from taking both arguments (i.e., the no-miracles argument and the pessimistic meta-induction) seriously. With respect to the case of the transition in nineteenth century optics from Fresnel's elastic solid ether theory to Maxwell's theory of the electromagnetic field, Worrall argues that:

> There was an important element of continuity in the shift from Fresnel to Maxwell, and this was much more than a simple question of carrying over the successful empirical content into the new theory. At the same time it was rather less than a carrying over of the full theoretical content or full theoretical mechanisms (even in approximate form)...There was continuity or accumulation in the shift, but the continuity is one of form or *structure*, not of content (1989, p.117; emphasis added).

These reflections led Worrall to believe that we should not accept scientific realism, which asserts, among other things, that the nature of the unobservable objects that cause the phenomena we observe is correctly described by our best theories. However, neither should we be antirealists in science. We should rather adopt 'structural realism' and epistemically commit ourselves *only* to the structural content of our theories. Since, according to Worrall, there is retention of structure across theory-change, structural realism both (1) avoids the force of the pessimistic meta-induction (by not committing us to believe in the theory's description of the 'furniture of the world') and (2) does not make the success of science (e.g., the novel predictions of mature physical theories) seem miraculous (by committing us to the claim that the theory's structure, over and above its empirical content, describes the world) (Ladyman, 2014).

Structural realism can be divided into an 'epistemic' and an 'ontological' variant. 'Epistemic structural realism' (ESR) is the view that "all we can know about the unobservable world is its structure" (Frigg and Votsis, 2011, p.232). 'Ontological structural realism' (OSR) can be characterized by the belief that structures are ontologically basic; i.e., "what originally appeared as the structure of something with unknown qualitative features is actually all that there is to nature" (Frigg and Votsis, 2011, p.260). In this paper I will focus solely on ESR.

Regarding ESR another distinction between two positions can be made. The first position is called 'direct epistemic structural realism' (DESR), and holds that "we can have fully-fledged knowledge (i.e., structural and non-structural knowledge) about the observable parts and aspects of the world ... but that our knowledge of the unobservable is only structural" (Frigg and Votsis, 2011, p.232). The other position is called 'indirect epistemic structural realism' (IESR), and holds that "we only have direct epistemic access to our sense data, our perceptions or at any rate something sensory, and only through them do we have (indirect) access to the world itself"(Frigg and Votsis, 2011, p.232). In this paper I will take the position of IESR. Hence when talking about 'ESR', I shall mean 'IESR' (however, the arguments put forward in this paper can, with some minor adjustments, be applied to DESR as well). Also, I will often use the term 'structural realism' when strictly speaking I should say 'ESR'.

The aim of this paper is to provide a model-theoretic account for structural realism. The paper is structured as follows. In section 1 I will introduce the so-called 'hierarchy of models'. The basic idea of the hierarchy of models is that via a hierarchy of theories, each viewed as a collection of mathematical structures (the 'models' of the theories), the 'shared structure' between theories at different layers of the hierarchy can be formally analysed. In the ideal case the theories at the various levels are isomorphic, in case of which we can infer that the 'high-level theory' (i.e., the highest level in the hierarchy of models) shares the structure of the 'phenomena' (i.e., the lowest level in the hierarchy of models).

In section 2 I will introduce different 'stances' towards the hierarchy of models. We will see that there is an empirical stance and a realist stance and that the two differ in how they close the gap between the 'data models' (another level in the hierarchy of models) and the 'phenomena'. I will show that, roughly put, the empiricist stance deals with this issue by identifying the two, thereby trivially taking the 'data models' to represent the 'phenomena'. In the absence of any justification for identifying the 'data models' and the 'phenomena', I (like Brading and Landry (2006)) shall take this move to be illegitimate.

We will see that this ultimately leaves the structural empiricist without an account for the structure of the 'phenomena'. Such an account is, however, of crucial importance for any model-theoretic analysis of the 'shared structure' of a 'high-level theory' and the 'phenomena' to be able to take place. In section 3 I will introduce a model-theoretic formalization of 'structural empiricism' in which the identification of the 'data models' and the 'phenomena' can be clearly recognized. This formalization will form the basis for a characterization of structural empiricism that *can* be accommodated in the hierarchy of models.

Our first objective will then be to develop an account for the structure of the 'phenomena'. This will lead us to section 4, in which I will introduce Bertrand Russell's causal theory of perception. In section 5 I will show how, based on

Russell's theory, we can account for the structure of the 'phenomena'. In section 6 I will show how given the structure of the 'phenomena' obtained through the application of Russell's theory, we can accommodate structural empiricism in the hierarchy of models. In section 7 I will show that the combination of Russell's theory and structural empiricism naturally leads one to a structural realist account. We will see that, given the addition of the level of 'external world' to the hierarchy of models, structural realism can be accommodated in the thus obtained 'extended hierarchy of models'. Then, given that both structural empiricism and structural realism are accommodated in the same framework (i.e., the extended hierarchy of models), we are in the position to formally analyse certain relations between the two. In section 8 I will show how the extended hierarchy of models can be even further extended in order to accommodate the 'problem of theory-change'.

Having said that, I will start by introducing the 'hierarchy of models'.

1 THE HIERARCHY OF MODELS

In this paper I will apply the so-called 'semantic view of scientific theories'. According to this view, scientific theories are to be " ... *characterized as* a collection of models [i.e., certain mathematical structures, see Hodges (2013)] that *share the same kind of structure*, and *the kinds of objects* that the *theory talks* about can be *presented* as positions in such models" (Brading and Landry, 2006, p.573; original emphasis). Models that share "the same kind of structure" are models that share certain logico-mathematical properties. More on this later. By objects in theories being presented "as positions in such models", Brading and Landry mean that objects are to be characterized only by what can be said of the shared structure of the models they feature in, that is, "by their [i.e., the objects] being instances of the same kind of structure (2006, p.572).

The semantic view of scientific theories can be used to account for the applicability of scientific theories to phenomena (Brading and Landry, 2006, p.573-

574). This idea was introduced by Suppes (1960, 1962). Suppes (1962, p.255) pointed out that scientific theorizing consists of a "hierarchy of theories and their models" that is supposed to bridge the gap between, on the one hand, a "high-level theory" and, on the other hand, the "lower level phenomena" that the theory is intended to talk about (Brading and Landry, 2006, p.574). Another way to put this is that there is a theory, characterized by a collection of mathematical structures (i.e., the theory's 'models'), associated with each layer in the hierarchy of models so that the relationship of shared structure between each layer can be formally analysed and experimentally evaluated (Brading and Landry, 2006, p.574).

Given such a hierarchy of models, the formal analyses of the applicability of a 'theory' to the 'phenomena' can proceed by model-theoretic methods. The analyses are made by appealing to the (non-)existence of isomorphisms between the various layers in the hierarchy. In this case the notion of 'isomorphism' is supposed to formally express the claim that theories have the same kind of structure (Brading and Landry, 2006, p.574).

Following Suppes (1962, p.253), Brading and Landry point out that there are at least two 'theories', next to the 'high-level theory' under consideration, required in order to connect the 'high-level theory' to the 'phenomena'. These are (1) an 'experimental theory of data' and (2) an 'empirical theory of the phenomena'. Together, the 'high-level theory', the 'experimental theory of data' and the 'empirical theory of the phenomena' make up the layers of the hierarchy of models (see Figure 1).

I do not intend to say much more about the fundamentals of the hierarchy of models. Doing so would lead us too far away from the aims of this paper. For an excellent introduction to this topic, I refer to Brading and Landry (2006) and Suppes (1962). For our purposes it is sufficient, yet crucial, to be aware of two points. The first being the following. If the 'experimental theory of data' is not explicitly formulated, then one can characterize such a theory by the 'collections of its models' (with 'models' taken to be in the 'usual' tarskian

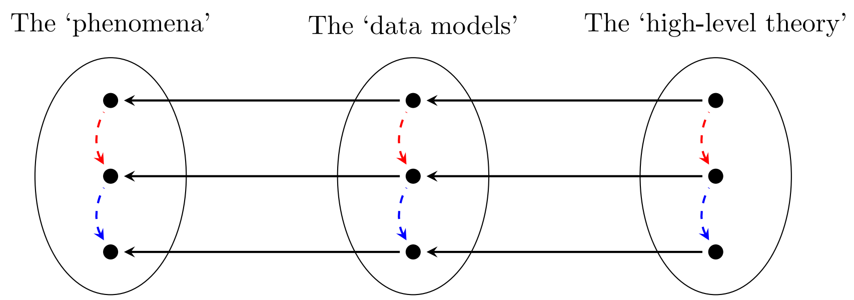

Figure 1: The hierarchy of models

sense). So, in that case, one's formal analysis begins with certain 'collections of models' which one uses to define the 'experimental theory of data' (Brading and Landry, 2006). This is merely a matter of definition. For more on the nature of the 'experimental theory of data', I refer to Suppes (1962).

The second point is more subtle. To connect the 'experimental theory of data' to the 'phenomena', one must establish that their models have the same kind of structure (Brading and Landry, 2006). However, as Brading and Landry (2006, p.575) point out, without an 'empirical theory of the phenomena', one cannot speak of the 'structure of the phenomena' (remember the definition of the semantic view of theories). And, without being able to speak of the structure of the phenomena, one surely cannot characterize the 'structure of the phenomena' in terms of the shared structure of its models. Hence Suppes remains silent on the issue of why we should suppose that 'models of data' have the same structure as the 'phenomena' (Brading and Landry, 2006, p.575). We will look closely at this issue in the next sections.

Before we proceed to the next section, I have to make one remark regarding the terminology to be used in this paper. I will use the following terms interchangeably: (1) 'high-level theory' and 'theory', (2) 'experimental theory of data', 'models of data' and 'data models', and (3) the 'empirical theory of the phenomena' and the 'phenomena'. This is allowed due to the semantic view of theories I will apply throughout this paper. Furthermore, I will use single quotation marks around a term to emphasize that I am talking about a level in the hierarchy of models (i.e., the 'phenomena' etc.).

2 DIFFERENT STANCES TOWARDS THE HIEARCHY OF MODELS

Given the hierarchy of models as introduced in the previous section, Brading and Landry (2006) offer three 'stances' to account for the applicability of a 'high-level theory' to the 'phenomena': (1) a methodological stance, (2) an empirical stance and (3) a realist stance. These stances differ in how they make the 'theory-world connection'; i.e., in how they close the gap between the 'data models' and the 'phenomena'. According to (1), the issue is irrelevant since we should simply start with the 'data models' and make no commitments regarding the relation between the 'data models' and the 'phenomena'. According to (2), we can say that the 'high-level theory' structures the 'phenomena' into 'data models', thereby imposing structure upon the 'phenomena' and turning the 'data models' into representations of the 'phenomena'. According to (3), we may say that what structures the 'phenomena' is the external world.

The importance (for the empirical stance and the realist stance) of the 'phenomena' having structure cannot be overstated. For suppose that the 'phenomena' would not have structure. Then one cannot formalize (by means of model-theoretic methods) the treatment of the structure of the 'phenomena'. This would imply that one could not use the account of 'shared structure' underlying the hierarchy of models in order to account for the applicability of a 'theory' to the 'phenomena' (Brading and Landry, 2006, p.575). In other words, it would make the application of model-theory to account for the applicability of a 'theory' to the 'phenomena' impossible.

While the connection between the models of a 'theory' and the 'data-models' can be accounted for solely in terms of what Brading and Landry (2006, p.573) call 'presentation' of shared structure (such 'presentation' being sufficient for non-representing theoretical entities), the connection between 'data models' and the 'phenomena' demands the addition of an account of 'representation'. The latter should account for the issue of how a physical theory that talks about theoretical kinds of objects, comes to be about particular objects (Brading and

Landry, 2006, p.576). Brading and Landry hold that appeals to shared structure are not up to this task (2006, p.576), for "[t]he question of the reality of particular physical objects cannot be settled semantically, that is, cannot be settled merely by appeal to a tarskian notion of a model: it depends crucially on some extra-semantic process whereby the connection between *what we say* and *what there is* is both established and justified" (Brading and Landry, 2006, p.576; original emphasis). This is what Brading and Landry mean when they say that "an account of representation is required" (2006, p.576) in order to connect the 'data-models' to the 'phenomena'. We will return to this issue in section 4 in which I will introduce such an account of representation.

Brading and Landry (2006) believe that in order to move from the methodological stance (which makes the fewest assumptions) to either the empirical stance or the realist stance, we need reasons to do so. In particular, if we are to adopt either of these alternatives, we need " ... a justification for the claim that data models share the same structure as the phenomena and, as a result, that the former can be taken as representations of the latter" (Brading and Landry, 2006, p.578). Let us look at the justifications (or lack thereof) offered by respectively the empirical stance and the realist stance

According to Brading and Landry (2006, p.578), the empirical stance holds the view that the 'high-level theory' structures the 'phenomena' into 'data models'. In this regard, 'data models' are " ... far removed from 'mere descriptions of what is observed', that is, from what we might call 'the phenomena"' (Brading and Landry, 2006, p.574). This view appears too in Van Fraassen's account of 'structural empiricism', for he suggests that we should simply *identify* the 'phenomena' with the 'data models', for "the data model is, as it were, a secondary phenomenon created in the laboratory that becomes the primary phenomenon to be saved by the theory" (Van Fraassen, 2002, p.252). By adopting this view, the step from the 'data models' to the 'phenomena' is made almost trivially: the 'data models' act as the "phenomena to be saved", so all we need to connect the 'theory' to the 'phenomena' is a guarantee of any shared structure by the ‘theory’ and the ‘data models’.

Bradley and Landry (2006) claim, and I agree with them, that this empiricist version of scientific structuralism avoids the question of *why* it should be assumed in the first place that the ‘phenomena’ can be identified with the ‘data models’. This is due the absence of an account of representation on the part of the structural empiricist. Also, I agree with Bradley and Landry in believing that, in any attempt in which one aims to move beyond the methodological stance, it is necessary to provide an account of what would allow us to connect the ‘data models’ and the ‘phenomena’. Hence, given the absence of such an account on the part of the structural empiricist, her position (i.e., the identification of the ‘data models’ and the ‘phenomena’; which is an extreme form of connecting the ‘data models’ and the ‘phenomena’) is unjustified. This conclusion poses a problem for the structural empiricist. More on this in the next section.

Now the realist stance. Bradley and Landry point out that structural realists, who adopt a realist stance and therefore presume that the external world structures the ‘phenomena’, frequently invoke the no-miracles argument to account for shared structure between the ‘data models’ and the ‘phenomena’. For given that not such shared structure would be present, the success of science would be a miracle (Bradley and Landry, 2006, p.579; see the Introduction for an account of the no-miracles argument). Furthermore, structural realism, insofar as it takes the ‘data models’ to share the structure of the ‘phenomena’, is, thus, committed to the claim that the kinds of objects presented by a ‘theory’ accurately represent the structure of particular objects of which “the world is claimed to consist” (Bradley and Landry, 2006, p.579). The ESR-ist says that with respect to these objects, all that can be known is that they are instances of structural kinds given by our ‘theory’; all that can be *known* is their structure (Bradley and Landry, 2006, p.579). Bradley and Landry conclude by saying that structural realism’s “claim that structural properties play a representational role at all is justified *entirely* by appeal to the no-miracles argument”

(2006, p.579; emphasis added). I have emphasized the word 'entirely' for this paper is an attempt to provide such a justification that is different from the no-miracles argument. Also, note that the no-miracles argument is by itself no account of *how* the external world structures the 'phenomena'; it is solely an argument for the necessity of this being the case.

Note what we have established so far. In the previous section, I introduced the hierarchy of models. In this section I introduced the different 'stances' towards the hierarchy of models. The stances differ in their views on how to bridge the gap between the 'data models' and the 'phenomena'. I have said (roughly put) that the empirical stance deals with this issue by identifying the two, thereby trivially obtaining an account of representation. By providing no justification for this move, I (like Brading and Landry (2006)) see no reason to accept it. The realist stance, on the other hand, holds that the 'phenomena' are structured by the external world. Their main argument for this view is the no-miracles argument.

3 STRUCTURAL EPIRICISM PART I

In this section I will introduce a model-theoretic formalization of the empirical stance called 'structural empiricism'. This formalization will show very clearly the views of the structural empiricist. Furthermore, it will show the sense in which the structural empiricist identifies the 'data models' and the 'phenomena' and the problem this leads too.

Having said that, I will now turn to the model-theoretic formalization of structural empiricism. I will introduce the formal framework as developed by Bueno (1997, 2000). Although Bueno (1997) uses his methods to account for structural empiricism, he admits that, "these patterns can be taken both by realists and empiricists in the articulation of their respective proposals" (2000, p.293). This will become important in later sections, in which I will show how to accommodate Bueno's framework in a model-theoretic account for structural realism.

Here follows the characterization of structural empiricism that Bueno tries to capture in his model-theoretic framework:

> To present a theory is to specify a family of structures, its *models*; and secondly to specify certain parts of those models (the empirical substructures) as candidates for the direct representation of observable *phenomena*. The structures which can be described in experimental and measurement reports we call *appearances*: the theory is empirically adequate if it has some model such that all appearances are isomorphic to empirical substructures of that model. (Van Fraassen, 1980, p.64; original emphasis)

Two points are worth mentioning. Bueno (1997) starts off his account by introducing the notion of 'empirical adequacy'. This notion is of great importance for the structural empiricist; i.e., "[i]t [i.e., 'empirical adequacy'] might be one of the most important concepts put forward to this kind of view [i.e., structural empiricism] (p.586). Hence Bueno aims to give an adequate formalization of this notion. Also, Bueno, likes us, applies the semantic view of theories, and doing so " ... leads quite naturally to a particular formulation of the notion of empirical adequacy" (Bueno, 1997, p.588).

Now his formalization proceeds as follows. Let us say that a specific theory T is characterized by the theoretic model $T =< T', Rn_i^T >$ with $i \in I$, where T' is a particular domain, Rn_i^T is a family of n-place relations and I is an index ranging over the natural numbers. In this setting, an empirical substructure E of T is a substructure of T satisfying the following conditions: (1) $E =< E', Rn_i^E >$ with $i \in I$, such that (2.a) $E' \subseteq T'$, and (2.b) $Rn_i^E = Rn_i^T \cap E'^n$ for each $i \in I$.

Next Bueno characterizes what calls the 'appearances' as a mathematical structure: $A =< A', Rn_i^A >$. The 'appearances' are "the structures obtained through experimental and measurement reports" (Bueno, 1997, p.589). Furthermore, Bueno takes the domain A' of A to be a particular set of "phenomenologically described objects" (1997, p. 594). More on this latter.

Now a theory T is said to be 'empirically adequate' if for some some of its models $T =< T', Rn_i^T >$, the 'appearances' A are embeddable in it; i.e., if there is a bijection $f : E' \rightarrow A'$ from an empirical substructure E of T into A, such that for every n-tuple $(x_1, x_2, ..., x_n)$ of elements of E', $Rn_i^E(x_1, x_2, ..., x_n)$ if, and only if, $Rn_i^A(f(x_1), f(x_2), ..., f(x_n))$. In other words, a theory T is empirically adequate if there exists an isomorphism between a particular empirical substructure E of T and the appearances A (Bueno, 1997, p.589).

So far the model-theoretic formalization of structural empiricism. We are now in the position to analyse the structural empiricist framework as put forward above. I will do so by analysing whether this framework can be accommodated in the hierarchy of models. In order for a framework to be able to be accommodated in the hierarchy of models, two requirements have to be met. First of all, each level in the framework has to be put (or at least *can* be put) in model-theoretic form. This requirement is due to the semantic view of scientific theories on which the hierarchy of models is based. Secondly, each level in the framework has to have a corresponding level in the hierarchy of models. Let's see whether structural empiricism meets these criteria.

Let's start by looking at structural empiricism's characterization of 'theory'. Since the 'theory' is put in model-theoretic form, there is no reason to suppose that it is incompatible with the level of the 'high-level theory' in the hierarchy of models. Hence we take the two to be compatible.

Now let's look at the 'appearances'. It can be seen that both Van Fraassen and Bueno identify the 'data models' and the 'phenomena', grouping them together under what they call the 'appearances'. This follows from the fact that Van Fraassen takes the 'appearances' to be "the structures which can be described in experimental and measurement reports" (van Fraassen, 1980, p.64) while Bueno characterizes the domain of these same 'appearances' as a particular set of "phenomenologically described objects" (1997, p.594). Now, Van Fraassen's characterization of the 'appearances' would surely fit the 'data models' of the hierarchy of models while Bueno's 'phenomenologically described

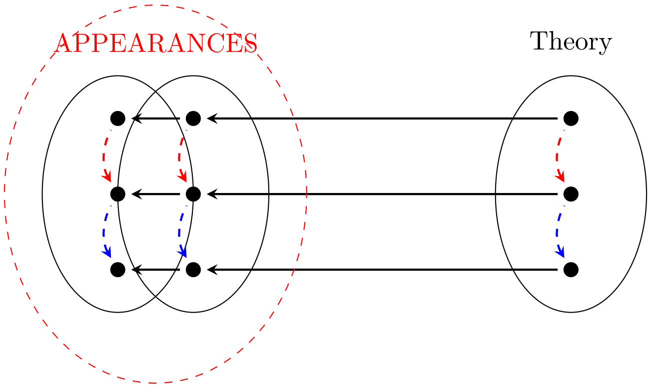

Figure 2: Structural empiricism

objects' would clearly belong to the domain of the 'phenomena'. Hence we have one and the same level (i.e., the 'appearances') that can be accommodated in two levels of the hierarchy of models (i.e., the 'data models' and the 'phenomena'). This can only mean that the structural empiricist does identify (from the perspective of the hierarchy of models) the 'data models' and the 'phenomena' (see Figure 2). Note that this is exactly what one would expect based on Brading and Landry's analysis (2006) of the empirical stance (see the previous section).

Remember that we established in section 2 that, given the absence of an account of representation (i.e., an account of how the 'phenomena' are represented by the 'data models'), there is no justification for identifying the 'data models' and the 'phenomena'. Hence, given the absence of such an account on the part of the structural empiricist, the two levels are *not to be identified*. This is problematic, for it was precisely *because of* the identification of the 'data models' and the 'phenomena', that the empiricist could account for the structure of the 'phenomena'. Now, given that this identification is unjustified, we have no reason to accept this account for the structure of the 'phenomena'. Hence we are left without an account for the structure of the 'phenomena'.

Another argument through which we reach the same conclusion is the following. The empiricist claims that the 'high-level theory' structures the 'phenomena' into 'data models'. Hence we are allowed to identify the latter two. But without any justification for the account that the 'high-level theory' structures the 'phenomena', there is no reason to suppose that the 'high-level theory' structures the 'phenomena'; let alone that the 'phenomena' are structured into the 'data models', as the structural empiricist believes. Hence we are left without an account for the structure of the 'phenomena'.

This conclusion is problematic for without an account for the structure of the 'phenomena', there is no reason to suppose that the 'phenomena', appearing as in the structural empiricist framework, can be put into model-theoretic form. Hence structural empiricism, as it stands, cannot be accommodated in the hierarchy of models (remember the requirements for being able to do so).

Another way to put this is as follows. The 'phenomena' having structure is a prerequisite for any model-theoretic analysis of 'shared structure' between the 'high-level theory' and the 'phenomena' to be able to take place. Hence, in the absence of an account for the structure of the 'phenomena', it is not clear if it would be possible, not even in principle, for the structural empiricist to apply model-theoretic methods to account for the applicability of a 'high-level theory' to the 'phenomena'.

Hence the structural empiricist is in need of an account for the structure of the 'phenomena'. This is what we will turn to in the next section.

4 RUSSELL'S CAUSAL THEORY OF PERCEPTION

In this section I will introduce Bertrand Russell's causal theory of perception. It might not be immediately clear what the use of this will be, but Russell's theory will turn out to be of crucial importance in (1) our account of the structure of the 'phenomena' (section 5) and (2) the structural realist account which is to be put forward in section 7. Having said that, let's look at the theory.

Bertrand Russell is generally seen to be an epistemic structural realist (Frigg and Votsis, 2011; Votsis, 2005). In his *The Analysis of Matter* (1927), he argues that we only have direct epistemic access to 'percepts'; the later being 'the basic units of our perception'. This makes Russell an IESR-ist (see Introduction). Like Votsis (2005), I will use the term 'percepts' without assuming any ontological scheme but rather to convey the idea that what is of importance is (loosely speaking) what we experience.

According to Russell, percepts lie at the end of causal chains originating in the external world. It is because of this relationship that percepts are taken to encode information about the external world (Votsis, 2005, p.1362). In order to reach this conclusion, Russell draws on a number of principles, the following two of which are especially important:

> *Helmholtz-Weyl Principle* (H-W): Different effects (i.e., percepts) imply different causes (i.e., stimuli or physical objects) (Russell, 1927, p.255).
>
> *Mirroring Relations Principle* (MR): Relations between percepts mirror (i.e., have the same logico-mathematical properties as) relations between their non-perceptual causes (Russell, 1927, p.252).

Armed with these principles, Russell deduces that from the structure of our perceptions we can " ... infer a great deal as to the structure of the physical world" (Russell, 1927, p.400). Or, to put it more precisely, Russell argued that, based on these principles, all we can guarantee is that the structure of our perceptions is at most isomorphic to the structure of the external world (Votsis, 2005, p.1362).

In order to assume H-W and MR, one, of course, has to give reasons justifying why it is that they can be assumed. While Newman (1928, p.142) speaks of Russell's causal theory as a theory allowing one to infer the structure of the external world from certain "plausible assumptions", he provides no reasons for the apparent plausibility of either H-W or MR.

Frigg and Votsis (2011) do a better job in doing so. For, they say, "[i]t is hard to imagine how we can interact with the world without accepting some version of these principles [i.e., H-W and MR]" (p.237). For suppose that, let's say, H-W did not hold at least most of the time. This would imply that the same (or sufficiently similar) stimuli repeatedly gave rise to different (or sufficiently dissimilar) perceptions for the same person. Recognizing a predator would then be a matter of luck, since the same (or sufficiently similar) stimuli, e.g. a tiger, would hardly ever give rise to the same (of sufficiently similar) perceptions, e.g. a tiger perception (p.237).

A similar justification can be given for 'W-H', the converse of H-W; i.e., the principle that "different stimuli/physical objects imply different percepts" (Frigg and Votsis, 2011, p.237). For suppose that different (or sufficiently dissimilar) stimuli repeatedly gave rise the the same (or sufficiently similar) perception in the same person. Correctly gauging changes in people's behaviour would hen be a miracle since different (or sufficiently dissimilar) behaviour, e.g. smiling versus crying, would hardly ever give rise to different (or sufficiently dissimilar) perceptions (Frigg and Votsis, 2011, p.237).

From reasons like these, Frigg and Votsis (2011, p.237) infer that " ... having a neurophysiology that functions in according with both H-W and W-H confers significant evolutionary and learning advantages". Hence it is plausible to assume (at least some version of) these principles.

One could proceed by providing even more fundamental reasons for the plausibility of H-W and W-H. However, doing so would lead us too far away from the aims of this paper.

Before we proceed, the following should be noted: in the aforementioned examples of Frigg and Votsis (2011), we assumed the existence of (something close to) an isomorphic relation between the 'phenomena' and the external world. As Frigg and Votsis point out (2011, p.237), an isomorphic mapping between percepts and stimuli would be the ideal case, but we can fall short of this ideal and still have knowledge. I will return to this issue in section 8. Also, as Votsis (2005), points out, " ... MR is a strong enough principle to guarantee inference at the isomorphic level" (p.1366) and it can be shown that that both H-W and W-H are derivable from it (Votsis, 2005).

5 THE STRUCTURE OF THE 'PHENOMENA'

In the previous section I have introduced Bertrand Russell's causal theory of perception. I have showed that according to Russell, perceptions lie at the end of causal chains which originate in the external world. I have introduced the so-called '*Helmholtz-Weyl Principle*' (H-W) and the '*Mirroring Relations Principle*' (MR). Based on these principles, Russell deduced that all we can guarantee is that the structure of the 'percepts' (i.e., Russell's basic units of perception) is at most isomorphic to the structure of the external world (Votsis, 2006, p.1362).

In this section we will put to use Russell's theory. Remember that, at the end of section 3, we came to the conclusion that the structural empiricist is in need for an account for the structure of the 'phenomena'. Up till now, the 'data models' and the 'phenomena' have been identified in what the structural empiricist calls the 'appearances'. We took this move to be unjustified. Hence the 'phenomena' and the 'data models' had to be distinguished. This left the structural empiricist without an account for the structure of the 'phenomena'. The importance of the structure of the 'phenomena' for both the empirical stance and the realist stance was argued for section 2.

In this section I will give an account for the structure of the 'phenomena'. I will start off from the assumption that the 'data models' and the 'phenomena' are to be distinguished. I will show how to account for the structure of the 'data models' (which will be easy) and the 'phenomena' (for which we will need Russell's theory). Based on this account, I will show (in section 6) that we are in the position to fully accommodate structural empiricism in the hierarchy of models.

Let's start off with the account for the structure of the 'data models'. As we established in section 1, we can formally characterize an 'experimental theory of data' by the 'collections of its data models' (i.e., the 'data models'). This is the characterization proposed by Brading and Landry (2006) and the characterization I will take too. Then, since the 'experimental theory of data' has been characterized by the 'collections of its data models', and since models, being mathematical structures, are structured (as exemplified in section 3), we can infer that the 'data models' are structured. Now take the domain of the 'data models' to be D', Rn_i^D with $i \in I$ to be a family of n-place relations on D', where I is an index ranging over the natural numbers. Now we have the mathematical structure $D =< D', Rn_i^D >$ representing the 'data models', hence characterizing the 'experimental theory of data'.

Before I proceed to the account for the structure of the 'phenomena', I first have to introduce two notions. The first of these is 'abstract structure', which is defined to be "an isomorphism class whose members are all, and only those, structures that are isomorphic to some given structure" (Votsis, 2005, p.1363). With regard to an isomorphism class, one can only identify what Russell calls the 'logico-mathematical' properties of its members (Russell, 1927, p.251-254). Logico-mathematical properties are properties like reflexivity, symmetry, transitivity etc. The notion of 'abstract structure' is to be contrasted with that of a 'concrete structure' (Redhead, 2001); whereas a concrete structure specifies *one domain of objects that comes with a set of relations*, an abstract structure just specifies *a constraint as to which domains of objects and relations qualify*; i.e., those domains equinumerous to some given number and those relations that share the same logico-mathematical properties (Votsis, 2006, p.1363).

Next, I follow Votsis (2005) in summarizing Russell's epistemic commitments (as introduced in section 4) as follows. Russell commits himself to the existence of: (1) concrete observational or perceptual structures, (2) abstract structures whose members are the concrete observational structures referred to in (1), and (3) the existence of concrete physical structures that (a) have as their domain members the external world causes of the domain members of (1) and (b) are members of the isomorphism classes referred to in (2).

Now a model-theoretic formalization of the structure of the 'phenomena' follows quite naturally. As observed by Demopoulos and Friedman (1985, p.624;

original emphasis) " ... on Russell's structural realism, of 'percepts' we *know* both their relations and structure". Furthermore we have, according to Russell, direct epistemic access to our percepts, the properties they have and the relations they stand in (Russell, 1927). It is then only natural to take Russell's percepts to constitute the domain of a model of the 'phenomena'. Let's take this domain to be P'. Note that, since we have specified the domain of objects of a model of the 'phenomena', it makes sense to define a set of relations on this domain. Now we take to take the properties and relations the percepts stand in (which they do; see Russell's theory) as the relations defined on this domain. Take these relations to be Rn_i^P. Now we have a mathematical structure $P =< P', Rn_i^P >$ representing the 'phenomena'. This makes any model of the 'phenomena' a concrete structure. Another way to put this is that any mathematical structure representing the 'phenomena' is fully interpreted.

In the remainder of this paper I will assume the existence of only one model of the 'phenomena'. This is a matter of convenience and could be changed if deemed necessary without posing any fundamental issues.

One might have doubts regarding my move of taking the 'percepts' to be the domain members of the 'phenomena'. Here follows a justification for doing so. Remember that our aim is to accommodate structural empiricism in the hierarchy of models. It seems fair to suppose that Brading and Landry (2006) take the 'phenomena' to be 'mental' phenomena, like Russell, instead of 'physical' phenomena (see Woodruff (2013) for the distinction between 'mental' and 'physical' phenomena). For in their account, the empirical stance identifies the 'data models' with the 'phenomena'. Now suppose that the 'phenomena' would not be 'mental' phenomena. Then they would have to be (something similar to) 'physical' phenomena. This implies that the empiricist would compare the structure of a 'high-level theory' to models the domains of which consist of 'physical' phenomena; i.e., phenomena that go beyond the empirical data. This would surely exceed any commitment an empiricist is prepared to make. Hence Branding and Landry are likely to use the same notion of 'phenomena' as I do (i.e., 'phenomena' as 'mental' phenomena), making Russell's percepts a natural candidate for the domain of the 'phenomena's to consist of.

Note that by now we have managed to structure (by model-theoretic methods) the 'data models' and the 'phenomena'. We have done so as follows. I took the 'experimental theory of data' to be the 'collections of its data models', as proposed by Brading and Landry (2006). Next, we took the 'phenomena' to encompass Russell's 'percepts' and the corresponding properties and relations, giving us the mathematical structure $P =< P', Rn_i^P >$ representing the 'phenomena'. So by now we have distinguished the 'data models' and the 'phenomena' and put both into model-theoretic form, which allows us to accommodate structural empiricism in the hierarchy of models. This will be the topic of the next section.

6 STRUCTURAL EMPIRCISM PART II

Now, given that we have obtained an account for the structure of the 'phenomena' (i.e., Russell's causal theory of perception) and have in fact put both 'data models' and the 'phenomena' in model-theoretic form, we are in the position to accommodate structural empiricism in the hierarchy of models. Remember that doing so is important for it would allow the structural empiricist to apply the model-theoretic apparatus underlying the hierarchy of models to evaluate the applicability of a 'high-level theory' to the 'phenomena'.

I continue the analysis from where we left off in section 3. We proceed as follows. Take $g : D' \to P'$ to be a function (i.e., from the domain of the 'data models' into the domain of the 'phenomena'). Suppose that g is an isomorphism. Then, by the definition of an isomorphism, for every n-tuple $(x_1, x_2, ..., x_n)$ of elements of D' and every $i \in I$, $Rn_i^D(x_1, x_2, ..., x_n)$ if, and only if, $Rn_i^P(g(x_1), g(x_2), ..., g(x_n))$.

Now, take $h : E' \to D'$ to be a function (i.e., from the domain of 'empirical substructure' E of T into the domain D' of the 'data models'). Suppose that h is an isomorphism. Now, take $i = g \circ h$ to be the composition of g and h

(i.e., $i : E' \to P'$). From basic set theory we know that the composition of two isomorphic functions is itself an isomorphic function. Hence, since both g and h are isomorphic, i is an isomorphism too.

Next, suppose we take the structural empiricist's 'appearances' A (as defined in section 3) to be our D (i.e., the 'data models'). Then, since $h : E' \to D'$ is an isomorphism between 'empirical substructure' E of T and the 'data models' D, and D is A, it follows that A is embeddable in T. Hence, by the definition of empirical adequacy (see section 3), T is empirically adequate.

Now, suppose we take the structural empiricist's 'appearances' A to be our P (i.e., our 'phenomena'). Then the same reasoning applies: since $i : E' \to P'$ is an isomorphism between empirical substructure E of T and the 'phenomena' P, and P is A, A is embeddable in T. Hence T is empirically adequate.

We see that, irrespective of whether we take the structural empiricist's 'appearances' to be the 'data models' or the 'phenomena', the notion of 'empirical adequacy' and hence structural empiricism can now be fully accommodated in the hierarchy of models. For suppose that we take the 'appearances' to be the 'data models'. Then any relation between the 'data models' and the 'phenomena' becomes irrelevant for the notion of 'empirical adequacy' to apply to the 'high-level theory' under consideration. Now suppose we take the 'appearances' to be the 'phenomena'. In that case, the structure of the 'phenomena' is a prerequisite for structural empiricism, and its notion of empirical adequacy, to be fully accommodated into the hierarchy of models. For without the structure of the 'phenomena' the notion of 'empirical adequacy' (and thus the core of the structural empiricist programme) would remain undefined. It is due to Russell's causal theory of perception that we managed to put the 'phenomena' into model-theoretic form, hence allowing its accommodation in the hierarchy of models. Either way, structural empiricism can, given the account we put forward, be fully accommodated in the hierarchy of models.

One side note. One could think of the 'data models' as an abstraction of the measurement reports obtained through scientific research. In that case, one could think of the scientific process as follows: one puts forward the 'data models' that are isomorphic to the 'high-level theory' under consideration, and sees whether these are isomorphic to the 'data models' that are isomorphic to the 'phenomena'. If so, the 'theory' captures the structure of the 'phenomena'. Note that this is merely a manner of thinking about the 'data models' meant to help us in dealing with its abstract nature.

7 STRUCTURAL REALISM

In the previous section we have seen how to accommodate structural empiricism in the hierarchy of models. However, the aim of this paper was to account for structural *realism*; not structural *empiricism*. So we are not done yet.

Note that up till now, we have assumed neither H-W nor MR (as defined in section 4). In this section I will show how, by assuming these principles, we can obtain a model-theoretic framework that fully accommodates structural realism. Furthermore, I will show in what sense structural realism can be considered to be an extension of structural empiricism. We will proceed as follows.

First I will show how to extend the hierarchy of models (defined in section 1) by adding to it the layer of 'external world', thereby obtaining what we shall call the 'extended hierarchy of models'. Next I will show how structural realism can be accommodated in this extended hierarchy of models. Then, given that we have done this, we have managed to accommodate structural empiricism and structural realism in the same model-theoretic framework. This will allow to formally analyse the relations between the two. In order to be able to do all of the above, we first have to put the 'external world' into model-theoretic form. This is the issue we will turn to now.

It is important to realize that the connection between the 'phenomena' and the external world is of crucial importance in any realist account which holds the view that "the world structures the phenomena" (Brading and Landry, 2006, p.575). Furthermore, the connection is crucial for any account that claims that any relation between the structure of the 'high-level theory' and the external world can be evaluated.

Bertrand Russell's causal theory of perception provides us with an account of how the external world structures the 'phenomena'. As I have shown in section 4, Russell assumes the '*Mirroring Relations Principle*' (MR) in order to infer that "all we can guarantee is that the structure of our perceptions is at most isomorphic to the structure of the world" (Votsis, 2006, p.1362). It will be this principle, together with Russell's theory as incorporated in the 'phenomena' (remember the 'percepts'), that will allow the extension of the hierarchy of models required to accommodate tructural realism.

For convenience, let me repeat MR:

> *Mirroring Relations Principle* (MR): Relations between percepts mirror (i.e., have the same logico-mathematical properties as) relations between their non-perceptual causes (Russell, 1927, p.252).

First we have to settle on some terminology. Russell uses the notions of 'stimuli' and 'physical objects' interchangeably. As Votsis states (2006, p.1362), Russell considers the former as lying in causal chains that can be traced back to the latter. Hence inferring something from the former implies inferring something about the latter. Russell takes stimuli to be "the events just outside the sense-organs" (1927, p.227). This makes them different from the 'phenomena', the domain of which to took to consist of Russell's percepts, which are, as we established in section 5, mental in nature. I will use the term 'causes' (instead of 'stimuli' or 'objects') for 'causes' are being directly referred to in MR.

As we established in section 5, Russell epistemically commits himself to the existence of concrete physical structures that (a) have as their domain members the external world causes of the domain members of the concrete observational structures (the latter being the members of our P'; i.e., the 'percepts') and (b) are members of the isomorphism class to which the concrete observational or perceptual structures belong to (i.e., to which our P belongs).

Now suppose that we share Russell's epistemic commitments and, although this is already implied by sharing Russell's epistemic commitments, suppose that MR holds. Then we can infer, at least to some extent (more on this later), the relations between 'causes' from the corresponding relations between the 'percepts'. Now, take $W =< W', Rn_i^W >$ to be a concrete physical structure (as defined by Russell) and suppose that this W represents the 'external world'. Now, since MR holds, we can infer at most that the structure of the external world is isomorphic to the structure of our perceptions. Let's suppose that the structure of the external world is in fact isomorphic to the structure of our perceptions. Then we can infer that there exists an isomorphism between P and W; i.e., that there exists a bijection $j : P' \to W'$ of P onto W, such that for every n-tuple $(x_1, x_2, ..., x_n)$ of elements of P' $Rn_i^P(x_1, x_2, ..., x_n)$ if, and only if, $Rn_i^W(j(x_1), j(x_2), ..., j(x_n))$. In other words, we can infer that W is a member of the isomorphism class to which P belongs.

Now, take $k = j \circ i$ to be the composition of i (as defined in the previous section; i.e., $i : E' \to P'$ was the isomorphism between empirical substructure E of T and the 'phenomena') and j (as defined above). Now the same reasoning applies as in the case of g and h; i.e., since the composition of two isomorphic functions is an isomorphism, and since j and i are isomorphic functions, k is an isomorphism too. In other words, k is an isomorphism between E and W; i.e., an isomorphism between empirical substructure E of 'high-level theory' T and the 'external world' W. In other words, the 'external world' W can be embedded into the 'high-level theory' T.

In this case we say that T is, what we will call, '*structurally correct*'; i.e., the 'external world' can be embedded into T. I believe that, if a 'theory' is structurally correct, then we can rightfully say that the 'theory' captures the structure of the 'external world'. Or, as Worrall (1989, p.101) might say, that the 'high-level theory' has latched onto the "universal blueprint".

Given the above-mentioned model-theoretic characterization of the external world, we can add the 'external world' (defined above) to the hierarchy of models, obtaining a four-layered 'extended hierarchy of models' (see Figure 3). Now suppose that 'structurally correctness' (as defined above) is to the structural

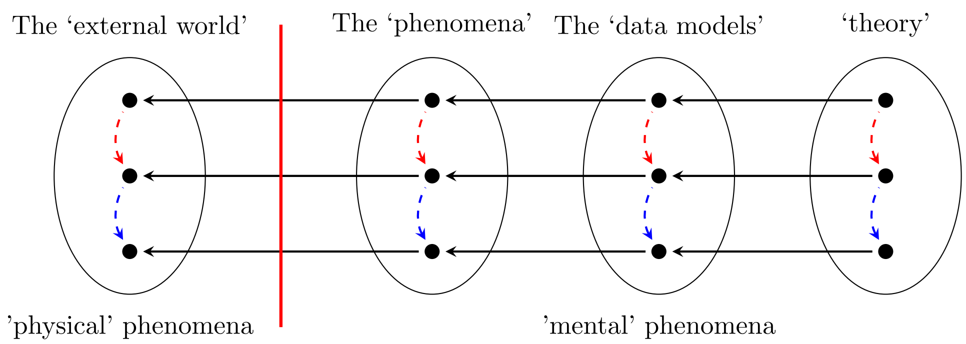

Figure 3: The extended hierarchy of models

realist what 'empirical adequacy' is to the structural empiricist. We know that the latter is one of the most important concepts put forward to structural empiricism (Bueno, 1997, p.586). So suppose that the same holds for structural correctness with regard to structural realism. Furthermore, suppose that, as we have established above, the notion of structural correctness can be accommodated in the extended hierarchy of models. Then it can reasonably be said that we have accommodated structural realism in our model-theoretic framework, i.e., in the extended hierarchy of models. It is in this sense that we have obtained a model-theoretic account for structural realism.

Now before we proceed, one remark is in order. As characterized, W is the concrete physical structure representing the 'external world'. However, all we can infer about W, based on MR, is that W is at most isomorphic to P. More than this cannot be claimed. We cannot single out one concrete physical structure from to the isomorphism class to which P belongs and say that *this* is the concrete physical structure representing the physical world. This is a logical consequence of the epistemic structural realist position according to which "physical objects themselves remain unknown in their intrinsic nature" (Russell, 1912, p.17) and according to which "[t]he only legitimate attitude about the physical world seems to be one of complete agnosticism as regard all but its mathematical properties" (Russell, 1927, p.270). Hence this limitation is consistent with indirect epistemic structural realism. In fact, it is a defining feature of it.

Note that, by now, structural empiricism and structural realism are accommodated in the same model-theoretic framework (i.e., the extended hierarchy of models). Hence any relations between the two, as reflected in the framework, can be formally analysed. Now it can be shown rather easily that, if MR establishes an isomorphism between the 'phenomena' and the 'external world', then it is the case that if a 'theory' is empirical adequate (see section 2) then it is structurally correct. However, when such an isomorphism is no longer assumed, this is no longer the case. On the other hand, the reverse (i.e., if a 'theory' is structurally correct, then it is empirically adequate) always holds. It is in this sense that structural realism can be considered to be an extension of structural empiricism; it has all the (formal) features of structural empiricism in addition to some features that have to do with the relation between the 'phenomena' and the 'external world'.

8 THEORY-CHANGE

Since we have accommodated structural empiricism and structural realism in the same framework, we can apply model-theoretic methods initially developed for structural empiricism (Bueno, 2000) and see how they relate to structural realism. One of the issues Bueno (2000) applies his model-theoretic methods to is the 'problem of theory-change'. As Bueno rightfully (2000, p.270) notes, " ... any minimally adequate interpretation of science has at some stage to come to terms with this problem". Hence so does structural realism.

The formalization of theory-change as formulated by Bueno (2000) is articulated in terms of an embedding. Let us consider two theories, $T1$ and $T2$, which are, as usual, formulated in terms of a collection of models. According to Bueno (2000, p.273) " ... the continuity across theory-change is achieved by embedding (the structures which characterise) $T1$ into certain models of $T2$"; i.e., for every model $M1$ of $T1$, there is a model $M2$ of $T2$ with a substructure $S2$ isomorphic to $M1$. Given this formalization, an instance of theory-change is characterized by an embedding of the models of a particular theory into some of the models of the theory succeeding it.

As pointed out by Bueno, viewing theory-change in this manner has a nice justification. For if two structures, S and S', are isomorphic, they are elementary equivalent; i.e., for every sentence α, α is true in S, if and only if, α is true in S'. Hence Bueno says, characterizing structural continuity as an embedding seems a natural move to make, since the latter immediately leads to an account of continuity throughout theory-change, namely: the preservation of the truth-values of the relevant claims by the respective theories (Bueno, p.273).

Note that the notion of continuity as defined here is *not cumulative*. After all, although sentences in different structures might be elementary equivalent, hence having the same truth-value, the extensions of the terms and relations in the structures (i.e., their 'interpretations') might differ. The idea is that 'conceptual change' can be accommodated by assigning different interpretations to terms and relations in $T1$ and $T2$. And given that $T2$ is conceptually and structurally 'broader' than $T1$, there is room for reinterpreting the concepts introduced by $T1$ in terms of those of $T2$ (at least for some models of $T2$) (Bueno, 2000). Now, Bueno notes, "[c]onceptual change is then thought of as interpretation change, whereas theory-change is accommodated in terms of structure preservation" (Bueno, 2000, p.274). The latter is, as mentioned already, accomplished by embedding the models of $T1$ into $T2$.

This formal framework, including its notions of 'conceptual change' and 'structure preservation', accommodates the idea that when we move, for instance, from Fresnel's theory of light to Maxwell's, there is preservation of structure (in terms of the of formal relations between the models of these theories; i.e., embedding the former in the latter), even though there can be high-level conceptual changes, which amount to changes in interpretation, as we move from one theory to another. Hence, although there is no longer any claim that light is a wave motion in the ether, we can still find structural similarities (in terms of the embedding model suggested) between the two theories (Bueno, 2000, p.274).

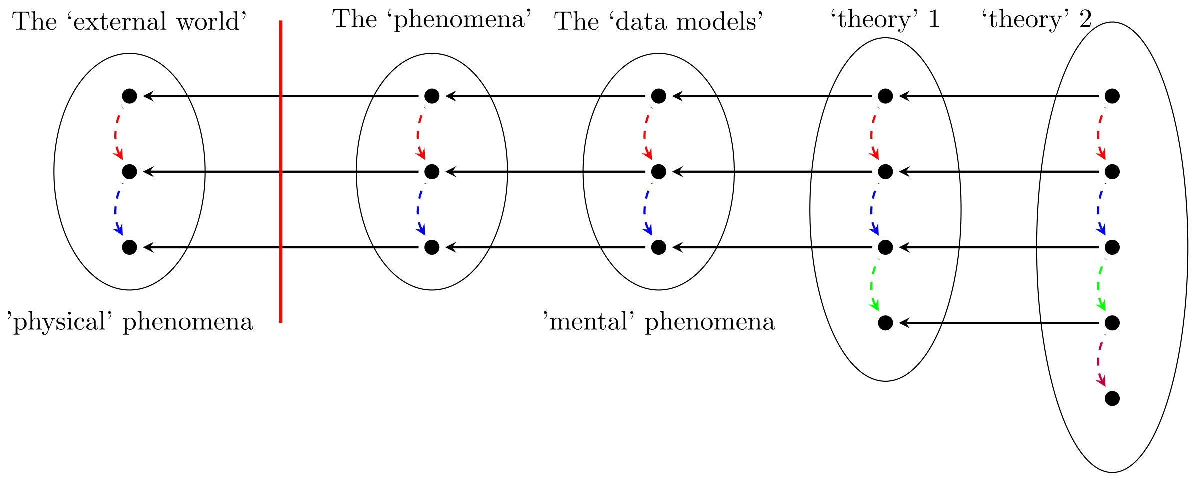

Figure 4: Theory-change

I want to make one remark. Since we have established (in the previous section) that empirical adequacy implies structural correctness (given that MR establishes an isomorphism between the ‘phenomena’ and the ‘external world’), it is easy to show (but I will not prove this here) that embedding a structural correct theory $T1$ into a replacing theory $T2$ implies that $T2$ is structurally correct as well. What this shows is that we could, in principle, extend the extended hierarchy of models even further by assigning different theories to consecutive layers in the hierarchy (see Figure 4). Then, like we do with the other layers of the hierarchy, we can evaluate the presence of shared structure among the various layers, ultimately for the sake of analysing whether or not a ‘theory’ shares the structure of the ‘external world’.

CRITICISMS

In this section I will respond to some criticisms one could raise against the views put forward the previous sections. I will respond solely to what I consider to be the most potent criticisms.

Before doing so, it is important to note that the model-theoretic framework put forward in the previous sections is intended to offer a rational reconstruction of scientific knowledge rather than a description of what goes on in science. Hence it should not be criticized solely on account of its divergence from actual scientific practice. Having said that, one could still object that the framework is unrealistically restrictive in the sense that I have assumed the existence of isomorphisms between the various layers of the extended hierarchy of models. I repeat that the existence of such isomorphisms is the *ideal*, but that scientific knowledge can still be obtained without their presence. Hence the non-existence of isomorphisms does not fundamentally undermine the account put forward in this paper. I want to call attention to the work of Bueno (2000), in which he uses the 'partial structures approach' (see Mikenberg et al., 1986; Da Costa and French, 1990) in order to account for partial preservation of structure in light of the the non-existence of isomorphisms.

Furthermore, Votsis (2006, p.1365-1366) points out that epistemic structural realism does not require a commitment to isomorphic relations between the 'phenomena' and the 'external world'. One could, for example, drop W-H. Doing so is not inconsistent with the views of the epistemic structural realist, for she can take isomorphic relations to be the ideal limits of knowledge. I refer to Votsis (2006) for an account of how epistemic structural realism can persist despite the fact that some of the assumptions made in this paper might not be met in practice.

Finally, I want to refer to Votsis (2003), who has cogent defence of ESR in light of the so-called 'Newman's Objection', thereby accounting for an issue (that Bueno (2008) believed to be) trivializing ESR.

CONCLUSION

In this paper I tried to give a model-theoretic account for structural realism. I started off by introducing the 'hierarchy of models' which can be used to formally analyse the structure shared by a 'high-level theory' and the 'phenomena'. Then I showed that structural empiricism (as characterized by Bueno (1997, 2000) and Van Fraassen (1980)) cannot be accommodated in the hierarchy of models.

I argued that this was ultimately due to the absence of an account for the structure of the ‘phenomena’.

Next I introduced Russell’s causal theory of perception and showed how this theory can account for the structure of the ‘phenomena’. Then I showed how, based on this account, structural empiricism *can* be accommodated in the hierarchy of models. Next I showed how, by the addition of the layer of ‘external world’ to the hierarchy of models, we could accommodate structural realism in the thus obtained ‘extended hierarchy of models’. Doing so provided us with the notion of a ‘high-level theory’ being ‘structurally correct’, which means that the ‘external world’ can be embedded into the ‘high-level theory’. The extended hierarchy of models allowed us to formally analyse the relations between structural empiricism and structural realism. Based on these analyses we saw that that the latter can be considered to be an extension of the former. Finally, I showed how the extended hierarchy of models can be extended even further by accommodating the ‘problem of theory-change’, thereby formally characterizing the notions of ‘conceptual change’ and ‘structure preservation’ (Bueno, 2000).

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